Final Report

Summary of Research

"Investigations into the Transient Analysis of System Reliability/Availability via Cumulant Functions"

Grant Number: NAG9-1547

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This document is the final report for grant number NAG9-1547 titled "Investigations into the Transient Analysis of System Reliability/Availability via Cumulant Functions". A summary of this work, including activities and findings, is contained herein. There are three appendices included with this report.

Research Activities

The period of performance of this work spans the annual period 1/13/03-1/12/04 [redacted] There was one student supported by this work, Martijn Kolloffel, for a period of 3 months in the summer of 2003. Martijn was an undergraduate student in the Industrial Engineering Department at New Mexico State University who had acquired previous background knowledge in the application of cumulant functions. Though he was an undergraduate student, he exceeded all other interested individuals in the evaluation criterion of grade point average, interest in work, and research potential. Maritijn worked under the direction of the PI of this work, Dr. Timothy I. Matis, who devoted 1.5 weeks of effort to this project. Though the allotted time for this project was limited, we were able to make many key research advancements through this project.

Martijn was directed to perform a preliminary literature search at the outset of this project to serve as a basis for procedural comparison. This literature search extended well beyond cumulant functions and generalized reliability networks into specific open

problems of current academic interest of which cumulant techniques may be beneficial. A summary of his literature search is included in Appendix A of this report.

The specific problem considered in this research is that of using cumulant functions to describe the reliability function of a system operating in a Markovian environment. The environment consists of non-fatal shocks that have the effect of weakening, though not destroying, the system on impact. Each shock has both an immediate reparable effect and a permanent non-reparable effect that increases the failure rate of the system in a non-linear fashion. These shocks may represent several events, most notably the failure of other system components as was originally described in the proposal. A graphical depiction of this model is given in Figure 1, where node 1 denotes the cumulative number of shocks, node 2 the current number of shocks, and node 3 the state of system functionality. The intensity functions are given by the functions f and the initial conditions for each node are given by $X_1(0)=X_2(0)=0$ and $X_3(0)=1$. Using established methods, i.e. Kolmogrov equations, obtaining the reliability function for such a system, i.e. Pr{System Functioning} = $E[X_3(t)]$, will approach computational intractability as the order of non-linearity increases and/or as the size of the system increases. The use of cumulant functions is a novel approach to this problem. Our work focused on how this approach may be applied, the accuracy of the approximation under various cumulant truncation levels, and the computational efficiency of this method. Our findings are summarized in the paper "Reliability Systems Subject to Non-Fatal Shocks" by Timothy I. Matis and Martijn Kolloffel that was recently submitted to the 2004 annual conference of the Institute of Industrial Engineering. This paper is included as Appendix

B of this report and was recently accepted by the conference for publication in the proceedings.

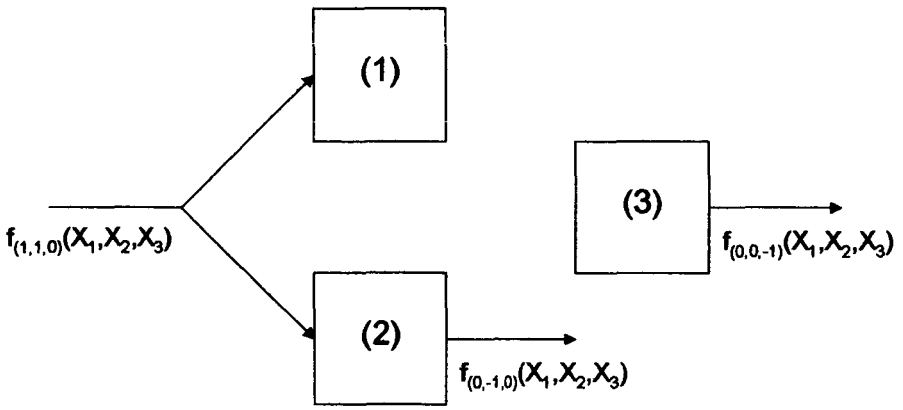

Figure 1. Graphical Representation of Reliability System

The research findings described in the attached paper may be summarized as follows: 1) Cumulant-based methods may be applied to this reliability model in a straightforward fashion, 2) the accuracy of the reliability functions approximations do not deviate greatly under varying cumulant truncation levels, 3) the robustness of the approximations increases with the order of cumulant truncation, and 4) the stability of the single system model provides evidence to the expandability of this procedure to large interacting systems. Empirical results were gathered using several "test-sized" models that described various types of state-dependencies. In particular, the polynomial intensity functions were varied in rate, order, and mixture of terms. It was observed that the cumulant method performed well in most for most models under the various truncation levels. The test models and numerical results of are contained in the paper of Appendix B for further reference.

The PI of this work also attended two research workshops sponsored by the Quality Education for Minorities Network for principal investigators of awards administered by the Faculty Awards for Research program of NASA. The first of these was in Albuquerque, New Mexico in October 2002, and the second in Atlanta, Georgia in October 2003. Representatives from NASA, past grantees, and representatives from other federally supported funding agencies presented at this conference. These conferences provided invaluable networking opportunities and insight into the current research needs of NASA. This helped guide the direction of this work and provide direction into future research pursuits that may arise from this work.

Dissemination Activities

As previously noted, the culmination of our experiments led to the drafting of a paper that has been accepted by the leading conference of the Industrial Engineering profession. This work will be presented at this conference which is to be held in May 2004. We hope to stimulated discussion amongst other researchers in stochastic processes and reliability theory towards the collaborative continuation of this research area. The PI of this work additionally traveled to Texas A&M University in January 2004 to meet with leading researchers about this work, most notably Dr. Tom Kiffe in the Department of Mathematics. Dr. Kiffe has several years of experience implementing cumulant-based computational routines in Mathematica®. We discussed several methods

through which our reliability-based programs, also authored in Mathematica, may be optimized in a computational sense.

Mentoring Activities

As part of this work, the PI met regularly with the supported student to discuss research progress, his future plans, and future research directions. We had scheduled meeting on Monday afternoons, yet would also meet in an unscheduled fashion nearly everyday. I shared with Martijn a document published by the Quality Education for Minorities Network titled, "Pathways to the Ph.D." in which useful information about the reasons and procedures for pursing graduate work are described. Though Martijn did not plan on attending graduate school at the outset of this project, he changed his mind and is now a Masters student in our program. An attitudinal survey of the evaluation of the mentoring activities of the PI through this period was administered to Martijn and is given in Appendix C of this document. The results of this survey were not made available to the PI of this work until this research grant had expired. A mentoring webpage is currently under construction by the

Appendix A

Literature Review
Martijn Kolloffel
NASA Project – Summer 2003

Cassady, C.R., Iyoob, I.M., "Analysis of Equipment Availability under Varying Corrective Maintenance Models". Work Review (2002)

In this paper, the authors present availability measures of repairable equipment. Different corrective maintenance models are compared and the effects of varying parameters of the models are examined. The effect of changing corrective maintenance models on equipment availability is analyzed. The point availability is the major measure of performance, which is illustrated in the paper by various graphics. Results are obtained using a simulation model created in Visual Basic, and a 95% confidence interval is obtained by multiple repetitions. The models of the impact of repair include Perfect Repair, Minimal Repair, Kijima Types Repair, and Quasi-Renewal Repair. The simulation model is developed to obtain functional approximations to the availability function for each of these models. This research does not provide any analytical solution methods, but it can be used as a tool when measuring the relative error of alternative analytical methods.

Dieulle, Laurence, "Reliability of several component sets with inspections at random times" European Journal of Operational Research 139 (2002) 96-114

This paper considers a random process representing a system of components with constant failure rates and subjected to inspections at times defining a renewal process. An analytical method for calculating the reliability function, its Laplace transform and the mean time to failure is given. These formulas are only computable if the Laplace transform of the inter-arrival law of the renewal process is explicit. The asymptotic behavior of the reliability and the failure rate of the system are studied. The reduced ability to model a variety of distributions, and the intractability when modeling large-scale systems however makes this study less applicable in reality.

Kijima, M. "Some results for repairable systems with general repair" Journal of Applied Probability 26 (1989) 89-102

In this paper Kijima develops general repair models for a repairable system by using the idea of the virtual age process of the system. Two models are constructed depending on how the repair affects the virtual age process. These models are then used to obtain an upper bound for the expected value of the survival function when a general repair is used. The introduction of the Virtual Aging Process is interesting especially considering the large number of references that have been made to this paper. Some knowledge of the survival function makes modeling maintenance policies easier, and the idea of virtual aging can greatly contribute to this endeavor.

Lam, Y., Tony, H, K., "A general model for consecutive-k-out-of-n: F repairable system with exponential distribution and (k-1)-step Markov dependence." European Journal of Operational Research 129 (2001) 663-682

In this paper, a general model for a repairable system in which the lifetime of a component depends on the number of consecutive failed components that precede the component. The failure and repair time are both exponentially distributed in this model. A transition density matrix is determined, and a general Markovian model is used to obtain some measures of performance such as the availability, the rate of failures and the reliability. The assumption is that a failed component after repair will be "as good as new". This publication has some general ideas that crossover to the other references presented here, and to my own field of research. The proposed methods however lack the ability to solve systems that show failure and repair times that are not exponentially distributed. It exemplifies that there is a serious need for analytical methods that can provide measures of performance for larger scale systems with non-exponentially distributed random variables.

Matis, J.H., Kiffe, T.R., "Stochastic Population Models, a compartmental perspective"

A classic book of Jacquez on compartmental analysis inspired this publication in which stochastic compartmental analysis is reviewed and generating functions are used to obtain many of the results. The theoretical development of the methods used is found in Chapters three and nine, for all practical purposes I focused on chapter three. This chapter explains the use of moments and cumulants when describing single population stochastic models. A standard approach for solving probability functions known as the Kolmogorov differential equations is illustrated. Cumulant functions are shown to be very useful for finding a distributions cumulants by obtaining and solving partial differential equations for associated generating functions. An immigration death model is used to illustrate the theoretical development of the generating function approach. The use of cumulant functions presented in this analysis is applied in the arena of reliability by Matis and Feldman, as described above. I have extended these ideas to find practical application in reliability systems with the support of Dr. T.I.Matis.

Matis, T.I., Feldman, R.M., "Using Cumulant Functions in Queueing Theory" Queueing Systems 40, (2002) 341-353

This publication demonstrates a new procedure for obtaining measures of performance of state-dependent queueing networks. This procedure uses cumulant generating functions and relates these to the intensity functions in the network. The service rate is expressed as polynomial function of the state of the system, from which a partial differential equation of the cumulative generating function is obtained. This partial differential equation then yields a set of ordinary differential equations, which are then solved to obtain the first and second moments of the system with Markovian arrival and service rates. The first and second moments of the random variables in the system provide transient information when describing the system. As the network increases in size this solution method remains tractable, in contrast to the method proposed by Rehmert in the dissertation reviewed above. Cumulant functions have previously been used by Matis and Kiffe to obtain measures of performance of ecological models, such as the spread of honeybees or muskrats. This cumulant derivation procedure is also used in the research of reliability systems subject to non-fatal shocks.

Pham, H. Wang, H. "Imperfect maintenance" European Journal of Operational Research 94 (1996) 425-438

The maintenance of deteriorating systems is often imperfect, as studied by using simulation in the first reference by Cassady and Iyoob. Imperfect maintenance studies using mathematical models for estimating availability functions and reliability have undergone some breakthroughs that are discussed and summarized in this paper. Treatment methods for imperfect maintenance, such as the (p, q) and (p(t), q(t)) rules, the improvement factor method, virtual age method, shock model method, (□ □) rule, are examined and explained. Preventative maintenance policies, such as age-dependent, periodic PM, failures limit policy, sequential PM policy, repair limit policy, and multi-component systems can indicate what model needs to be selected. Although the focus of this paper tends towards modeling maintenance policies to reduce cost, the underlying principals can be very useful when thinking about reliability systems in a more general sense.

Rehmert, I. J., "Time-Dependent Availability Analysis for the Quasi-Renewal Process." Diss. Virginia Polytechnic Institute and University (2000)

This research is based upon the quasi-renewal process proposed by Wang and Pham which is an alternative to the widely studied imperfect repair model, the (p, q) model, proposed by Brown and Proschan. A quasi renewal process can realistically describe the behavior of repairable equipment. The framework provided in this dissertation allows for the description of the time-dependent behavior of this non-homogeneous process. Two equivalent expressions for the point availability of a system with operation intervals and repair intervals that deteriorate according to a quasi-renewal process are constructed. These expressions are used to provide upper and lower bounds on the approximated point availability. Laplace transforms are used to solve the resulting expressions. The quasi renewal function and the point availability function are found for exponential, normal, and gamma operating and repair intervals. It is necessary to truncate the expressions to invert to the time domain to obtain numerical results. The usefulness of this approach seems limited because it does not find exact expressions for all distributions, and the calculations seem to be intractable as the size of the system increases.

Scarsini, M. Shaked, M. "On the value of an item subject to general repair or maintenance" European Journal of Operational Research, Vol.122 (2000) 625-637

This paper introduces and studies finding a practical expression of the monetary value of an item. The model that is constructed takes the repair and the maintenance procedures that are applied to it during its lifetime. The number of repairs and the degree of the repair are taken into account when doing the calculations. The degree of the repair follows the ideas of virtual aging modeling as was presented earlier by Kijima in his virtual aging models. The uncertainty is introduced into the model by the distributions of the inter repair or inter maintenance periods. This paper shows how the virtual aging process can be used to model cost under repair and maintenance procedures.

Sheu, S., Griffith, W.S., "Multivariate Age-Dependent Imperfect Repair" Naval Research Logistics, Vol. 38 (1991) 839-850

This article considers models of systems whose components have dependent life lengths with specific multivariate distributions. Components are repaired according to a corrective maintenance scheme, meaning that components are repaired upon failure. Only two types of repair are considered in this paper: perfect repair, and imperfect repair. The study focuses on a model in which the nature of the repair is age dependent. The model uses the (p(t), q(t)) rule which was earlier described in this reference list in the publication by Wang. An expression for the cumulative hazard function is derived, which can be useful when describing reliability of systems. The paper however lacks any numerical examples that demonstrates finding the hazard function.

Wang, H., "A survey of maintenance policies of deteriorating systems" European Journal of Operational Research 139 (2002) 469-489

This survey summarizes, classifies, and compares various existing maintenance policies for both single-node and multi-node systems. All these models fall into categories such as: Age replacement policy, block replacement policy, periodic preventive maintenance policy, failure limit policy, sequential preventive maintenance policy, repair cost limit policy, repair time limit policy, repair number counting policy, reference time policy, mixed age policy, group maintenance policy, etc. The characteristics, advantages, and drawbacks for each kind of policy are addressed. Maintenance and replacement problems have been studied for the past several decades, and this invited review gives a clear overview of the models used. Wang's research has introduced the concept of "Virtual Age" when modeling Quasi-renewal or imperfect repair.

Appendix B

Reliability Systems Subject to Non-Fatal Shocks

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Abstract

The reliability modeling of numerous physical systems is critical in the prevention of system failure. In many instances, the failure rate of system components is a function of non-fatal shocks or stresses to the system that occur at discrete points in time. These shocks are assumed to be identical and reparable, and they impact the failure rate of the system in a non-linear fashion via the cumulative and current number of shocks. In this paper, we demonstrate the application of the cumulant derivation procedure to this reliability system in a Markovian environment. This approach utilizes a truncated cumulant generating function to generate a set of ordinary differential equations whose numerical solution approximates the reliability function. These approximations are obtained under various truncation levels whereby this approach is shown to be tractable for large systems.

1. Introduction

In this paper, we consider a single component operating in an environment of non-fatal reparable shocks. These shocks may represent a wide variety of events, including the failure of other system components, instantaneous system stresses, or the states of a compartmentalized wear process. The shocks are assumed to be homogenous and their arrival is governed by a stationary Poisson process. They are repaired according to an exponentially distributed infinite server repair process that begins immediately upon shock arrival whose rate is a function of the total number of cumulative shocks. The failure of the component is Poisson distributed with the rate being a function of both the current number of unrepaired shocks and the total number of shocks that have ever been received by the system. In other words, each shock has both an immediate reparable effect and a permanent weakening effect on the system. Let $X_1(t)$ and $X_2(t)$ be integer-valued random variables taking values in $[0,1,..,\infty]$ that denote the cumulative and current number of shocks at time t respectively. Let $X_3(t)$ be an integer-valued random variable taking values $[0,1]$ that represents the state of the component at time t, where $X_3(t)=1$ denotes a functioning component and $X_3(t)=0$ denotes a failed component. All possible unit changes that may occur in the state of the system in a small interval of time are contained in the set B and the corresponding state-dependent intensity (rate) functions will be denoted as $f_{(b1,b2,b3)}(X_1,X_2,X_3)$ for $(b_1,b_2,b_3)\in B$. This system is graphically depicted in Figure 1.

We assume that initially the component is operational, $X_3(0)=1$, and no shocks have been received $X_1(0)=X_2(0)=0$. The intensity function corresponding to component failure, $f_{(0,0,-1)}(X_1,X_2,X_3)$, is specified as a nonlinear function of the cumulative and current number of shocks, $X_1(t)$ and $X_2(t)$, and represents the damage process previously described. Our primary interest is in approximating the reliability function of the component, i.e. the expected value of $X_3(t)$, for all $t \geq 0$ using truncated cumulant generating functions. These approximations are compared to simulated values for several systems under various intensity function specifications and truncation levels.

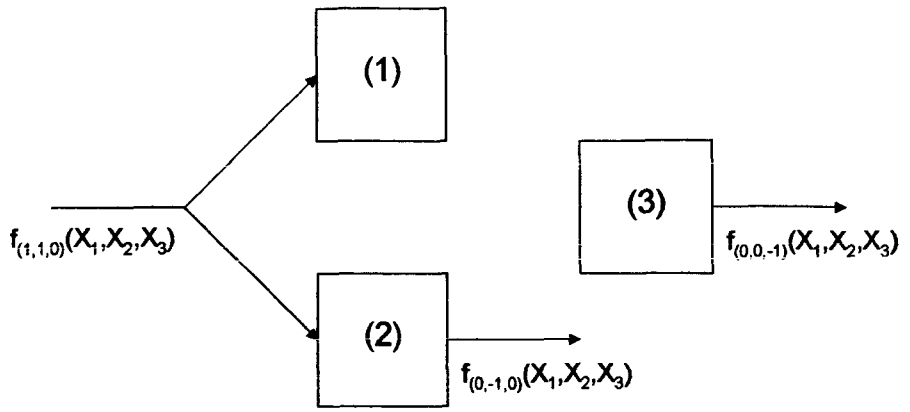

Figure 1: Graphical Representation of Reliability System

2. Cumulant Derivation Procedures

Let $X(t)=(X_1(t),X_2(t),,X_3(t),)$ be a random vector of the system state at time t. It follows that X(t) forms a Markov process with an absorbing state that denotes component failure. While such a process may be solved exactly using convention means, i.e. Kolomogorov Equations, such an approach is intractable for large systems or those with an infinite state space. As an alternative, the cumulants of the state-distribution of X(t) may be approximated using a truncated generating functions. These cumulant measures correspond directly to the common measure of mean, variance, covariance, skewness, etc. of the state distribution. Previous investigations into the nature of cumulant functions by Kendall[1][2] and Smith[3] reveal several interesting properties. In particular, the cumulants of a multivariate normal distribution greater than the second order are null, and the marginal cumulants of a Poisson distribution are equal. This and other properties of cumulants are exploited by this approximation procedure. This section contains only a brief overview of the cumulant derivation procedure based on the full development found in Matis and Feldman [4][5], Matis [6], and Matis and Kiffe [7].

Let $M(\theta_1,\theta_2,\theta_3,t)$ be the multivariate moment generation function of X(t) defined in the usual manner, and let $K(\theta_1,\theta_2,\theta_3,t)$ be the multivariate cumulant generating function defined as

$$K(\theta_1,\theta_2,\theta_3,t) = \sum_{a_1,\ldots,a_n \in N^+} \frac{k_{a_1,\ldots,a_n}(t)\theta_1^{a_1}\cdots\theta_n^{a_n}}{a_1!\cdots a_n!}$$

where N^+ denoted the set of non-negative integers. The joint cumulants $k_{a_1,\ldots,a_n}(t)$ of the system are defined as functions of the individual moments through the relationship between the generating functions, i.e.

$$K(\theta_1,\theta_2,\theta_3,t) = \ln\left(M(\theta_1,\theta_2,\theta_3,t)\right). \tag{1}$$

The moment generating function of X(t) related to the polynomial intensity functions of the system through a partial differential equation. This relationship was investigated by Bartlett[8] and Bailey[9] and thereby dubbed the "Random Variable Technique". While the specification of the partial differential equation of the moment generating is almost immediate, finding a solution is usually computationally intractable. The cumulant derivation procedure involves substituting a truncated cumulant generating function for the moment generating function into the partial differential equation according to Eq. (1). A set of approximating ordinary differential equations is obtained upon expanding the partial derivatives, substituting Taylor series expansions for the exponential terms, and equating the coefficients of

unique combinations of the θ_i's. The accuracy of these approximations is dependent upon the specification and order of the polynomial intensity functions and the level of cumulant truncation. Previous investigations, Matis[6], have shown that truncation at the 3rd order is generally sufficient for "good" approximations of the first order cumulants (mean), while that at the 4th order is sufficient for the marginal second order cumulants (variance). Further developments of the cumulant derivation procedure will be described in a reliability context.

3. Cumulant-Based Analysis of the Non-Fatal Shock Process

In this section, we demonstrate the application of cumulant-based procedures to the non-fatal shock process described in the introduction of this paper, see Figure 1. This will be shown for one instance of the problem under a unique set of intensity functions. Let the intensity function of the system be defined as

$$f_{(1,1,0)}(X_1,X_2,X_3) = \lambda$$
$$f_{(0,-1,0)}(X_1,X_2,X_3) = \mu_2 X_1^2(t)X_2(t)$$
$$f_{(0,0,-1)}(X_1,X_2,X_3) = \mu_3(X_3(t)+X_1(t)X_2^2(t)\ X_3(t))$$

where λ=.10, μ_2=.05, and μ_3=.025. In other words, the rate of repair is dependent upon the cumulative number of shocks and the failure rate of the component is dependent on both the current and cumulative number of shocks, both in an increasing manner. A partial differential equation of the moment generating function of X(t) is found using the "Random Variable Technique" as

$$\frac{\partial M(\theta_1,\theta_2,\theta_3,t)}{\partial t} = \lambda e^{(\theta_1+\theta_2-1)}\frac{\partial M(\theta_1,\theta_2,\theta_3,t)}{\partial\theta_1} + \mu_2 e^{(-\theta_2-1)}\frac{\partial^3 M(\theta_1,\theta_2,\theta_3,t)}{\partial\theta_1^2\partial\theta_2}$$
$$+\mu_3 e^{(-\theta_3-1)}\left(\frac{\partial M(\theta_1,\theta_2,\theta_3,t)}{\partial\theta_3}+\frac{\partial^4 M(\theta_1,\theta_2,\theta_3,t)}{\partial\theta_1\partial\theta_2^2\partial\theta_3}\right). \tag{2}$$

An mth order truncated cumulant generating function $K_m(\theta_1,\theta_2,\theta_3,t)$ and a Taylor series expansion of the exponential terms is substituted into Eq.(2) yielding the expression

$$\frac{\partial e^{K_m(\theta_1,\theta_2,\theta_3,t)}}{\partial t} = \lambda\left(\sum_{i=1}^{\infty}\frac{(\theta_1+\theta_2)^i}{i!}\right)\frac{\partial e^{K_m(\theta_1,\theta_2,\theta_3,t)}}{\partial\theta_1} + \mu_2\left(\sum_{i=1}^{\infty}\frac{(-\theta_2)^i}{i!}\right)\frac{\partial^3 e^{K_m(\theta_1,\theta_2,\theta_3,t)}}{\partial\theta_1^2\partial\theta_2}$$
$$+\mu_3\left(\sum_{i=1}^{\infty}\frac{(-\theta_3)^i}{i!}\right)\left(\frac{\partial e^{K_m(\theta_1,\theta_2,\theta_3,t)}}{\partial\theta_3}+\frac{\partial^4 e^{K_m(\theta_1,\theta_2,\theta_3,t)}}{\partial\theta_1\partial\theta_2^2\partial\theta_3}\right). \tag{3}$$

Expanding the partial derivatives, converting moments to cumulants (Smith[3]), and equating the coefficients of like polynomial terms on the right and left hand side of Eq. (3) yields a closed set of $\left(\frac{1}{3!}\prod_{i=1}^{3}(i+m)\right)-1$ ordinary differential equations. The size of the generated sets of ordinary differential equation does not permit their demonstration in this paper, yet the number of such equation is 9 under a truncation level of *m*=2, 19 under *m=3*, and 34 under *m*=4. Approximations to the low order cumulants are found upon numerically solving these sets of ordinary differential equations.

4. Numerical Results

The sets of ordinary differential equations generated from Eq. (3) were numerically solved using the mathematical software Mathematica® under the truncation levels *m=2, 3, 4*. These were then compared to simulated point estimates based on 10,000 replications using the software ProModel®. The initial

conditions for the process have all cumulants set equal to zero except for $k_{0,0,1}(0)$, which corresponds directly to the reliability of the component at time 0, is set to one. As previously noted, truncating the cumulants at m=2 implies that the state-distribution is normally distributed. Increasing the level of truncation to m=3 brings in skewness and m=4 brings in kurtosis, in addition to the effects of the higher-order cross cumulants. The cumulant approximations for the reliability of the component, i.e. $R(t)=E[X_3(t)]$, is given in Figure 2.

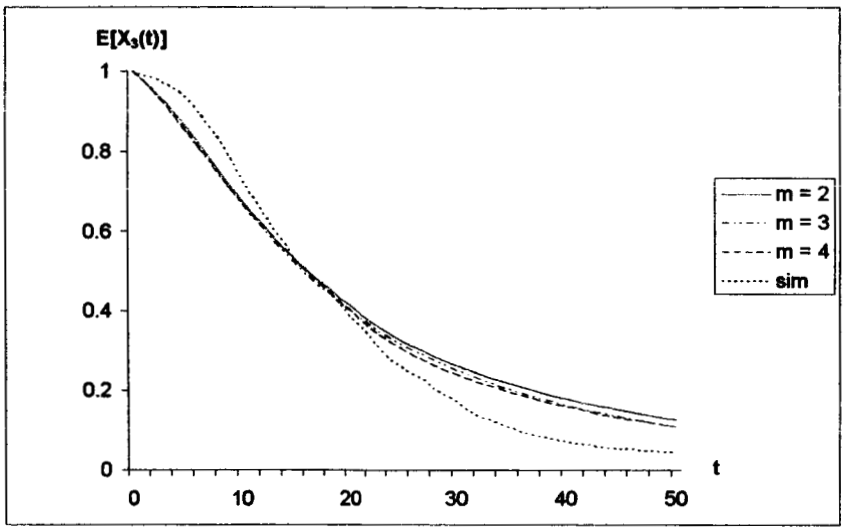

Figure 2: Graph of $R(t) = E[X_3(t)]$ for Varying Truncation Levels

The approximation of $R(t)=E[X_3(t)]$ is relatively tight between all values of m and close to the simulated value. This result is promising as the reliability function for systems subject to similar non-fatal shock processes may be well approximated using small values of m. As such, this result provides evidence that approach may be extended to similar large-scale networks in a computationally efficient manner. Though not of primary interest in this paper, the differences in approximations of the variance of the current number of shocks, $Var[X_2(t)]$, between truncation levels is noteworthy and is given in Figure 3.

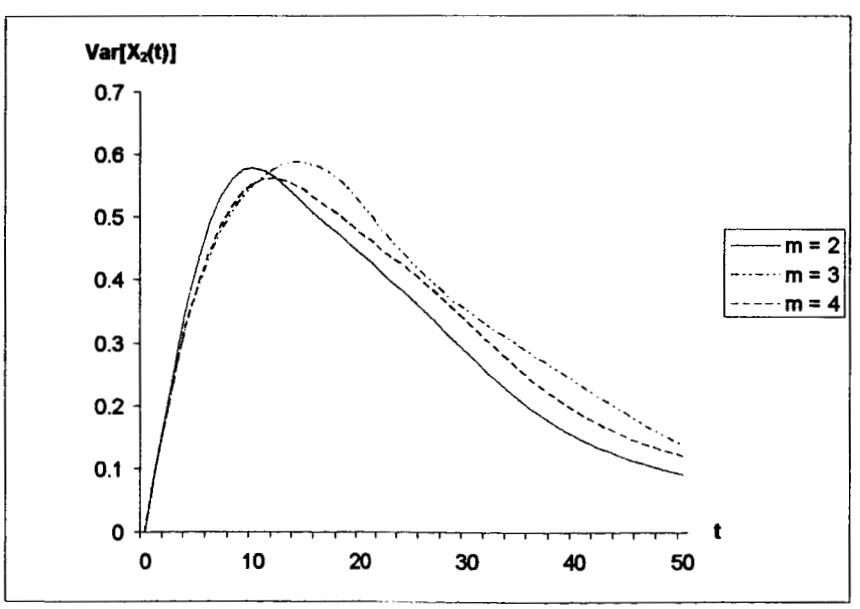

Figure 3: Graph of $Var[X_2(t)]$ for Varying Truncation Levels

The specified model was then evaluated under the parameter specification λ=.10, μ_2=.05, and μ_3 =.005 yielding the graph in Figure 4. Comparing the similarity of the cumulant approximations for various truncation levels in both Figures 2 and 4 provides evidence that the precision of the reliability function approximations do not significantly vary with the rate of system failure. Comparing these approximations to simulations, however, provides evidence that the accuracy of the approximations increases as the rate of system failure decreases.

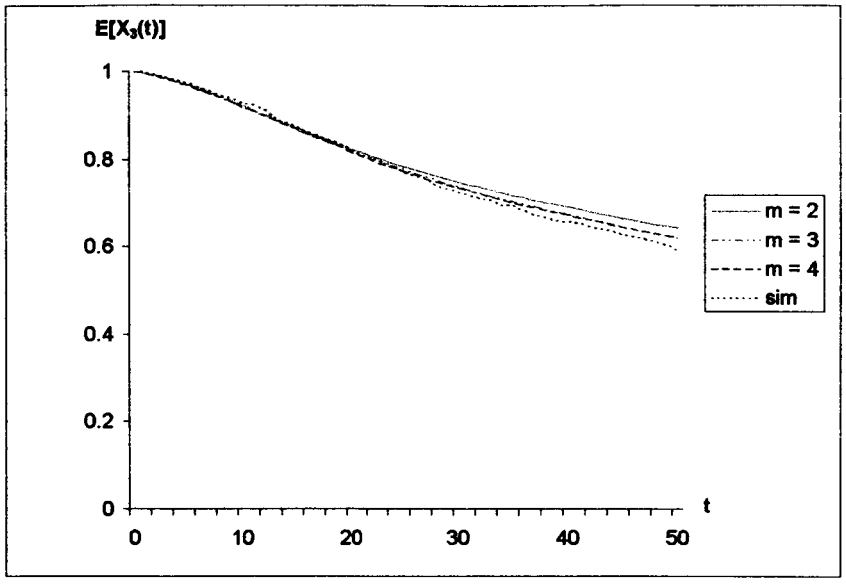

Figure 4: Graph of R(t) = $E[X_3(t)]$ for Varying Truncation Levels

The polynomial intensity function of the model were then redefined as

$$f_{(1,1,0)}(X_1,X_2,X_3) = \lambda$$
$$f_{(0,-1,0)}(X_1,X_2,X_3) = \mu_2 X_1^3(t)X_2(t)$$
$$f_{(0,0,-1)}(X_1,X_2,X_3) = \mu_3(X_3(t)+X_1^2(t)X_2^3(t)\, X_3(t))$$

increasing the dependency of the failure and repair rates on the state of the network. The graph of the reliability function is given in Figure 5. This deviation of the cumulant approximation under $m=2$ provides evidence that 2nd order truncation is not sufficient for approximating the cumulants of systems that are strongly state-dependent. This observed result is consistent with the previously stated properties of cumulants, i.e. truncation at $m=2$ assumes a multivariate normal state distribution. This normal assumption clearly does not hold for systems with strong state-dependency in which skewness is clearly present.

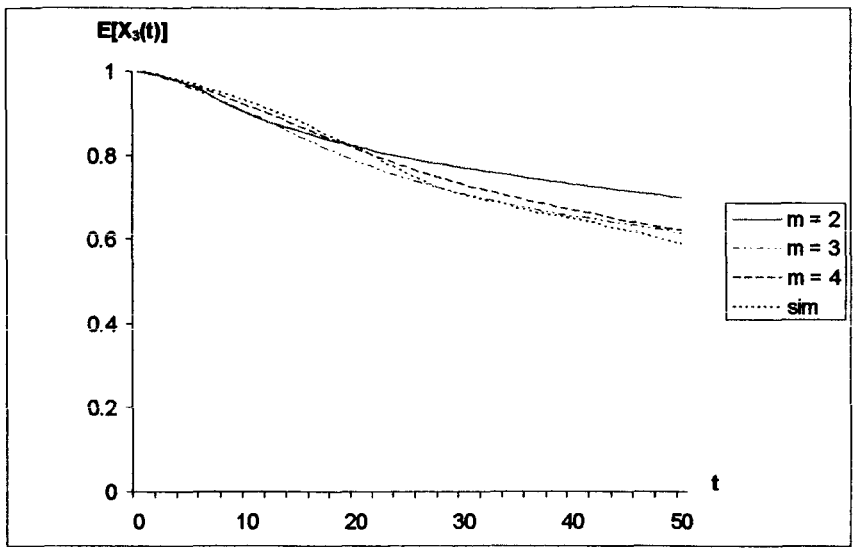

Figure 5: Graph of R(t) = $E[X_3(t)]$ for Varying Truncation Levels

5. Conclusions

In this paper, we have shown the application of cumulant derivation procedures to a reliability system subject to non-fatal shocks, i.e. a state-dependent reliability system. The effect of various truncation levels on the accuracy of the reliability function approximation was demonstrated for various parameters and intensity functions. The similarities between these approximations provide insight into the expandability of the approach to large, complex systems. A copy of the Mathematica® computational routines used to set up and solve the approximating set of ordinary differential equations may be obtained from the authors upon request.

6. Acknowlegements

This work was partially supported by the National Aeronautics and Space Association under Grant# NAG9-1492 and the Office of Naval Research under Grant# N00014-02-1-1040.

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Appendix C

Investigations into the Transient Analysis of System Reliability/Availability via Cumulant Functions
Student Evaluation -- Summer 2003 -- Attidudnal Survey
Please return survey to Adela Castro, Bldg. ECIII Room 201
CONFIDENTIAL -- Individual responses will not be disclosed to Dr. Matis until grant completion

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree	No Answer
1. I feel confident in my ability to use stochastic processes beyond this research project			✓			
2. This project has increased my overall knowledge of stochastic processes			✓			
3. This project has opened up new general areas of sceintific interest for me		✓		✓		
4. I feel that the research topic I worked on is in an area of scholarly importance	✓					
5. I feel that these research activities were well-balanced in theory and application			✓			
6. I feel that the research performed was practically useful to NASA			✓			
7. I feel that Dr. Matis has explained the technical details of the work in a clear manner		✓				
8. I feel that Dr. Matis cares about the progress I have made in this research area		✓				
9. I would feel comfortable talking with Dr. Matis about my academic concerns		✓				
10. I feel that Dr. Matis cares about my overall academic progress		✓				
11. I would feel comfortable talking with Dr. Matis about appropriate personal concerns		✓				
12. I feel that Dr. Matis cares about me as a person		✓				
13. I feel that Dr. Matis has exposed me to opportunities for graduate study		✓				
14. I feel that Dr. Matis has helped me in making future career decisions		✓				
15. Overall, I feel that Dr. Matis was an effective mentor		✓				

Short Answer:

Briefly comment on the things which were most enjoyable about this project

- Defining the problem was enjoyable. The process of defining the problem made me think in creative ways.
- Calculating results with mathematica and comparing to simulated results.
- Modelling in ProModel was enjoyable

Briefly comment on the things which were the least enjoyable about this research project

- Understanding and mastering the cumulative derivation method confronted me with some mathematical shortcomings, which I did not enjoy working through.
- The abstractness of some concepts involved decreased my enjoyment

Briefly comment on the mentoring that you received through participation in this project

- Dr. Matis is a great mentor, in all respects.
- Enforcement of deadlines, or implementation of a timeline helped managing my work, Dr. Matis gave me a lot of responsibility in organizing the research, which was at times difficult for me.

Additional comments

- I learned a lot about ~~dam~~ the process of doing research, in addition to learning what I like about stochastic processing, and what I dislike.
- The field of stochastics has many usefull implication and uses in industry and militairy applications, as is evident from the research paper produced during the research